Valence-bond entanglement entropy of random chains of non-Abelian quasiparticles

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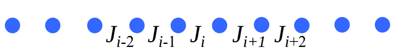
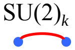
Introduction

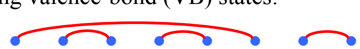
- Non-Abelian quasiparticles, described by $SU(2)_k$ Chern-Simons-Witten theory, may exist in certain FQH states (likely candidates are $\nu = 5/2$ and $12/5$).
- $SU(2)_k$ particles carry quantum numbers called topological charge, which can take values 0, ½, 1,…k/2. Topological charge is, in many ways, similar to ordinary spin.
- For a system of N $SU(2)_k$ particles, the dimensionality of the Hilbert space of states grows asymptotically as $\boldsymbol{d}^N$ where $\boldsymbol{d} = 2\cos(\pi/(k+2))$ is the quantum dimension of the particles. (For ordinary spin $\boldsymbol{d} = 2$, corresponding to $k \to \infty$).
- If two $SU(2)_k$ particles with topological charge ½ fuse, their total topological can be 0 or 1. A **singlet** is formed if they fuse to 0, a **triplet** if they fuse to 1.
- A simple Hamiltonian describing a chain of N *interacting* $SU(2)_k$ particles was introduced in [1],

$$H = -\sum J_i\, \Pi_i^0;\; J_i > 0 \qquad (1)$$

where J_i is singlet-triplet energy splitting of neighboring pairs of particles and Π_i^0 is the singlet projection operator (See Eq. (2) in next panel).



- The uniform model ($J_i = 1$) was studied in [1] while the random version (random J_i) was studied in [2].
- As for ordinary spin singlets, an $SU(2)_k$ singlet can be represented by a valence bond 
- Again, as for ordinary spin, the sector of zero total topological charge of the Hilbert space associated with a chain of $SU(2)_k$ particles is spanned by the set of non-crossing valence-bond (VB) states:



- The similarity between this representation of the Hilbert space of a chain of $SU(2)_k$ particles and that of ordinary spin-½ particles raises the question: Can the valence-bond Monte Carlo method introduced for spin-½ particles in [3] be used to study the random chains of $SU(2)_k$ particles described by Eq. (1)? The answer is "yes".

Valence-bond Monte Carlo

- The ground state $|GS\rangle$ of H can be projected out from an arbitrary state $|S_0\rangle$ [3]

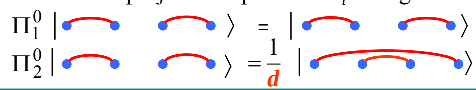
$$(-H)^n|S_0\rangle = \sum_{P_n} W(P_n)|S(P_n)\rangle \propto |E_0|^n |GS\rangle$$

with: $P_n = [a_1,\ldots,a_n],\; a_i = 1\ldots N$ refers to $\prod_{p=1}^n \Pi_{a_p}^0$ which is a term of $(-H)^n$,

and $|S(P_n)\rangle \propto \left(\prod_{p=1}^n \Pi_{a_p}^0\right)|S_0\rangle$

$$W(P_n) = \prod_{p=1}^n J_{a_p} w_{a_p};\; w_{a_p} = 1 \text{ or } 1/\boldsymbol{d} \qquad \text{[see Eq. (2)]}$$

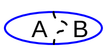
- Action of the projection operator Π_i^0 on a given VB state



(2)

Valence-bond entanglement entropy

- A quantum system composed of two parts A and B

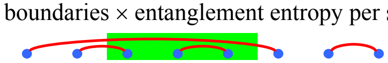


$\rho_A = \mathrm{Tr}_B\left[|GS\rangle\langle GS|\right]$ Reduced density matrix $\qquad S_A \equiv -\mathrm{Tr}[\rho_A \log_2 \rho_A]$ von Neumann (vN) entanglement entropy

- vN entanglement entropy of a spin in a spin-½ singlet is 1, while that for an $SU(2)_k$ singlet is $\log_2 \boldsymbol{d}$. [2]
- S_L is the vN entanglement entropy between a block of L particles with the rest of a chain of N particles.
- Ref. 2: $|GS\rangle$ of a *random* chain is frozen into a random singlet phase with

$$S_L = \frac{\ln \boldsymbol{d}}{3}\log_2 L + \text{Constant}; \quad \boldsymbol{d} = 2\cos\left(\frac{\pi}{k+2}\right) \qquad (3)$$

- The logarithmic scaling in Eq. (3) becomes that of the random Heisenberg chain as $k \to \infty$ ($\boldsymbol{d} \to 2$), and that of the random transverse Ising model for $k = 2$ ($\boldsymbol{d} = \sqrt{2}$). [4]
- For a given VB state, S_L = # of singlets crossing the block boundaries × entanglement entropy per singlet.



$S_L = 1 \times \log_2 \boldsymbol{d}$

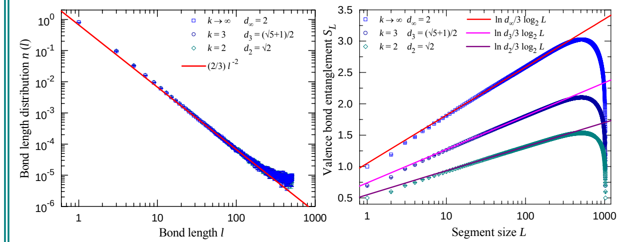
- For a given VB state, the bond length distribution $n(l)$ is defined as the probability for a bond to have the length l.
- From $n(l)$ of the $|GS\rangle$, which can be computed by VB Monte Carlo, the VB entanglement entropy is defined by [5]

$$S_L^{\mathrm{VB}} = \left(\sum_{l<L} 2n(l)l + \sum_{l>L} 2n(l)L\right) \times \log_2 \boldsymbol{d}$$

- VB entanglement entropy exhibits the logarithmic scaling [Eq. (3)] of vN entanglement entropy of spin-½ systems [5].

Monte Carlo results

- For each random configuration (a set of random J's), the state obtained by the decimation scheme described in Ref. [2] is chosen to be $|S_0\rangle$.
- $N = 1024$, $n = 10\,N$, 5000 random configurations.



- $n(l) = 2/(3l^2)$ as $1 \ll l \ll N$, independent of $\boldsymbol{d}$. [6]
- Valence-bond entanglement entropy exhibits the scaling (3) of the von Neumann entanglement entropy.
- The effective central charge, connected to the pre-factor of the entanglement entropy [see Eq. (3)], is $\ln \boldsymbol{d}$ as predicted. [2]

Conclusions

☑ Random chains of non-Abelian quasiparticles can be simulated by the valence-bond Monte Carlo method.

☑ The ground state of the random chain freezes into a random singlet phase.

☑ The predicted logarithmic scaling [see Eq. (3)] of the block entanglement entropy of the random chain is confirmed.

References

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