Valence-Bond Monte Carlo for Chains of Non-Abelian Quasiparticles

<u>Huan Tran</u> & Nick Bonesteel

Department of Physics & NHMFL

Florida State University

Work supported by US DOE

2008 APS March Meeting
New Orleans, March 13, 2008

Non-Abelian quasiparticles

- Exotic quasiparticle excitations in certain FQH states (possibly $\nu = 5/2$, 12/5), described by $SU(2)_k$ Chern-Simons theory with $k = 2, 3, \ldots$
- Quasiparticles carry a "topological charge" = 0, ½, 1 … $k/2$, which is in many ways similar to ordinary spin.
- Two particles with topological charge ½ fuse to the total topological charge 0 or 1, which we refer as *singlet* or *triplet*.
- *Key feature:* The Hilbert space associated with a system of N $SU(2)_k$ particles has dimension $\sim d^N$

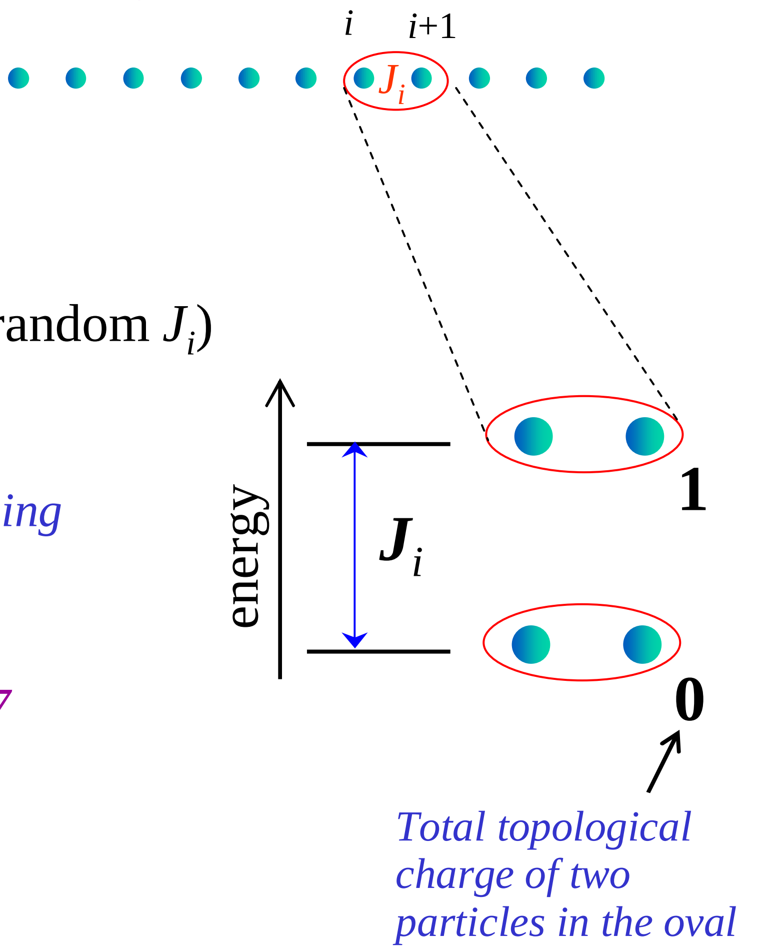
$$d = 2\cos\left(\frac{\pi}{k+2}\right)$$

Quantum dimension

Chains of $SU(2)_k$ particles

- *Hamiltonian* for a chain of N $SU(2)_k$ particles *(Feiguin et al., '07)*

$$H = -\sum_i J_i \Pi_i^0; \quad J_i > 0$$



- Π_i^0: projects sites i and $i+1$ onto a singlet
- *Uniform chains* ($J_i = 1$) *& Random chains* (random J_i)

k	***d***
2	$\sqrt{2}$
3	$\phi = \frac{(\sqrt{5}+1)}{2}$
4	$\sqrt{3}$
⋮	⋮
∞	2

→ *Transverse Field Ising Model (TFIM)*

→ *Golden Chains*
Feiguin et al., '07

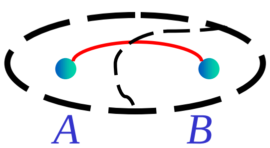
⋮

→ *Heisenberg Chains*

Valence-bond representation

- *An $SU(2)_k$ singlet*: a valence bond (VB) connecting two particles:

An $SU(2)_k$ singlet = 

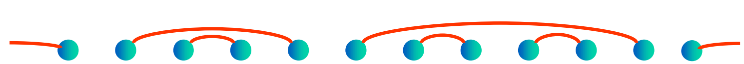


$$\rho_A = \mathrm{Tr}_B\left(|GS\rangle\langle GS|\right)$$ *Reduced density matrix*

$$S_A \equiv -\mathrm{Tr}\left(\rho_A \log_2 \rho_A\right)$$ *Entanglement entropy*

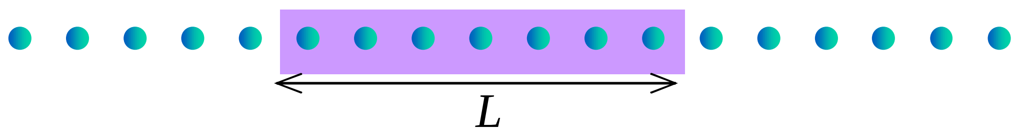
- *Key difference*: The entanglement entropy per bond of $SU(2)_k$ singlets is $\log_2 d$ instead of *1* for spin-½ singlets *(Bonesteel and Yang, '07)*

- *Key similarity*: States with total topological charge zero can be represented as superpositions of non-crossing VB states:



Block entanglement entropy

- *Block entanglement entropy* S_L is the entanglement entropy of a block of L particles with the rest of a chain of N particles



$$H = -\sum_i J_i \, \Pi_i^0$$

- For $1 << L << N$: $S_L = \frac{c}{3} \log_2 L + \text{Constant}$

- *Uniform* chains: $c = 1 - \frac{6}{(k+1)(k+2)}$ *Feiguin et al., '07*

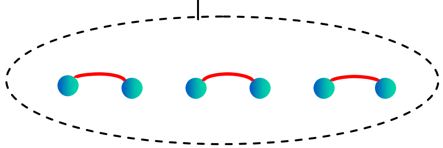
- *Random* chains (RSRG):
  - Heisenberg chain ($k \to \infty$): $c = \ln 2$
  - TFIM ($k = 2$): $c = \frac{1}{2} \ln 2$ — *Refael and Moore, '04*
  - All values of k: $c = \ln d$ — *Bonesteel and Yang, '07*

- Monte Carlo calculation.

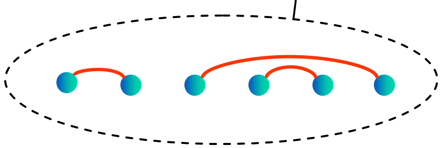
VB Monte Carlo

- Sample the $|GS\rangle$ from the VB basis
- *Idea*: project out the ground state by repeatedly applying $(-H)$ to some initial VB state $|S_0\rangle$ *(Sandvik, '05):*

$$(-H)^n|S_0\rangle = \left(\sum_{i=1}^{N} J_i \Pi_i^0\right)^n |S_0\rangle = \sum J_{i_1}\cdots J_{i_n}\left(\Pi_{i_1}^0\cdots\Pi_{i_n}^0\right)|S_0\rangle = \sum_{\alpha} W(\alpha)|\alpha\rangle \propto |\mathrm{GS}\rangle$$



Initial VB state $|S_0\rangle$

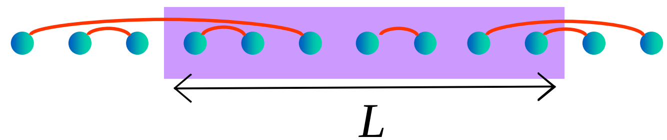


All non-crossing VB states $|\alpha\rangle$ appear in the sum

- Weight factor $W(\alpha)$: easy to compute and update for Monte Carlo
- Straightforward to generalize to $SU(2)_k$ particles

VB entanglement entropy

- *Single* VB state $|\alpha\rangle$: VB entanglement entropy = ‘True’ entanglement entropy

$|\alpha\rangle =$ 

$$S_L^{\alpha} = \boxed{\text{Number of bonds coming out the block of size } L} \times \boxed{\text{Entanglement entropy per bond}}$$

- VB entanglement entropy of $|\mathrm{GS}\rangle$ as a *superposition* of single VB states: $\sum_\alpha W(\alpha)|\alpha\rangle$

$$S_L^{\mathrm{VB}} = \frac{\sum_\alpha W(\alpha)\, S_L^{\alpha}}{\sum_\alpha W(\alpha)}$$

Alet et al., ‘07
Chhajlany et al., ‘07

- *Random spin-½ chains* ($k \to \infty$): VB entanglement entropy shares many features with the ‘true’ entanglement entropy, e.g., logarithmic scaling:

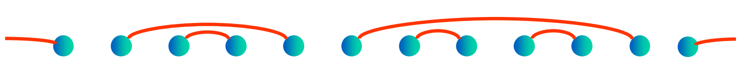
$$S_L^{\mathrm{VB}} = \frac{c}{3}\log_2 L + \text{Constant}$$

Alet et al., ‘07

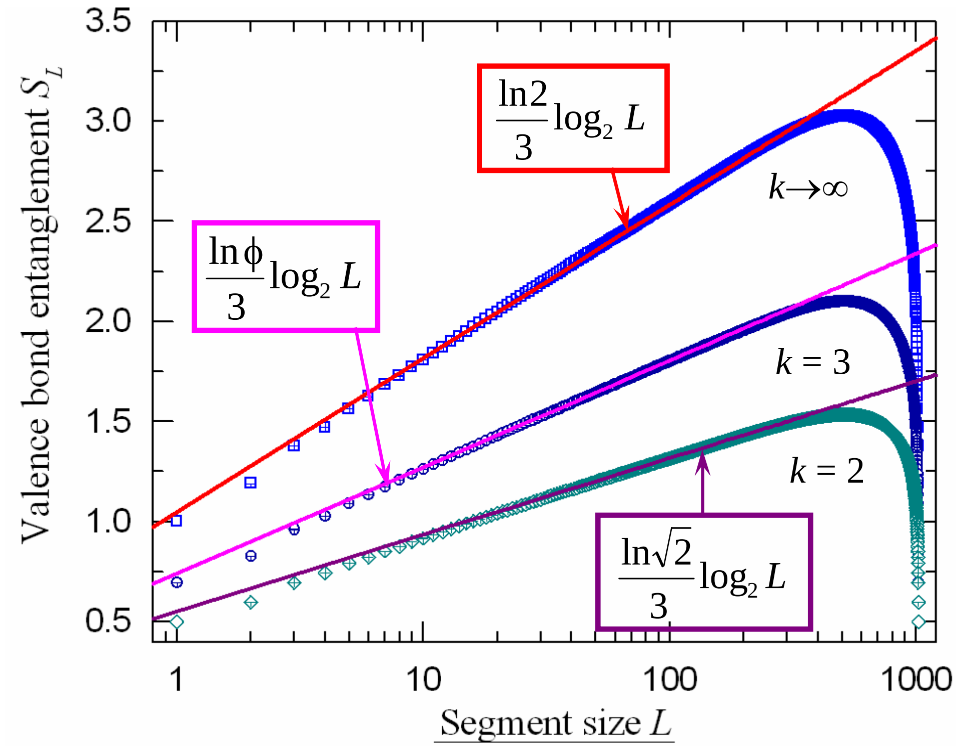
- *Random chains of $SU(2)_k$ particles*: TRUE for any k!

Results: VB entanglement entropy

- *Random chains*: Excellent agreement between the VB entanglement entropy and the 'true' entanglement entropy.
- *Reason*: $|GS\rangle$ freezes into a *random singlet phase* at long length scales *(Fisher, '94)* $\Rightarrow$ $|GS\rangle \approx$ a single VB state.

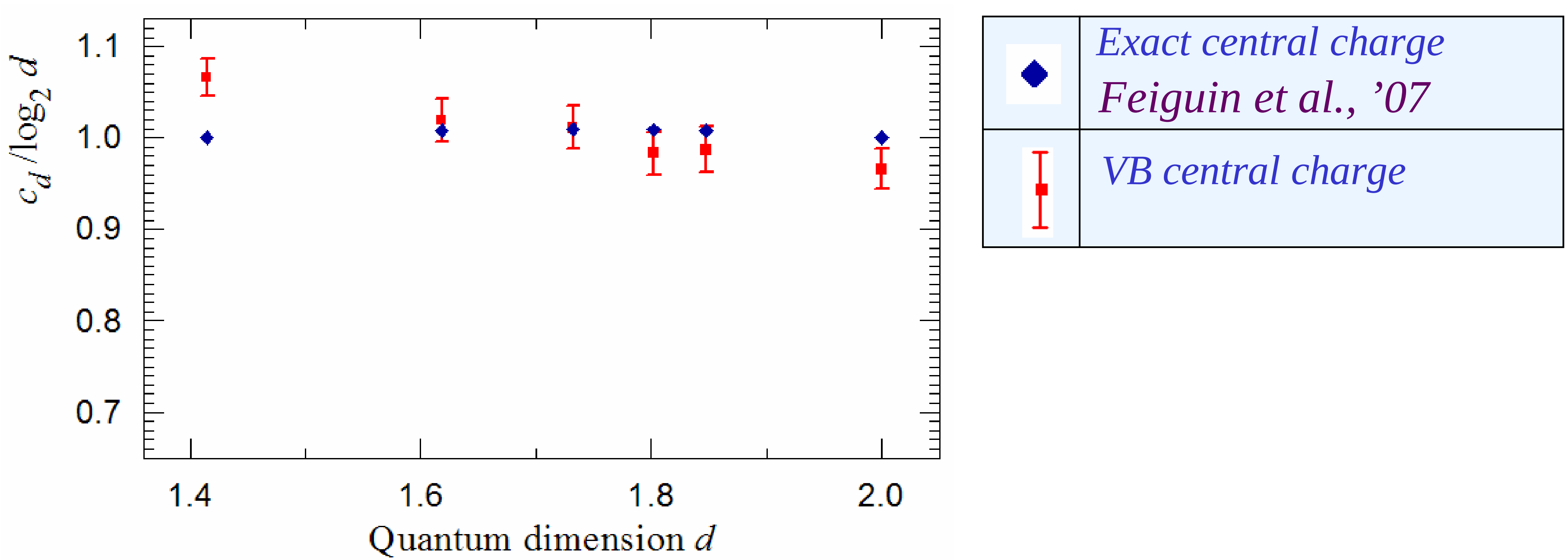


Random chains



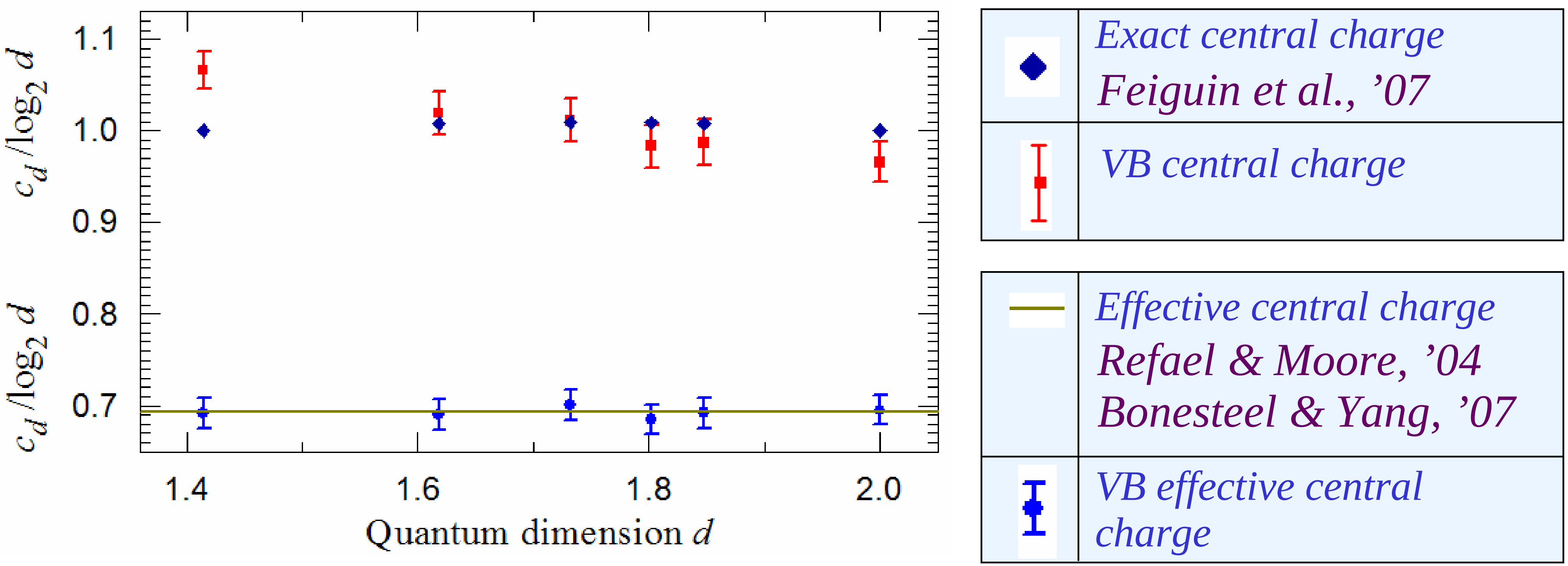
- *Uniform chains*: the $|GS\rangle$ is a superposition of many VB states.
- No reason to expect VB entanglement entropy and 'true' entanglement entropy to be the same.

Results: VB effective central charge



- *Valence-bond (effective) central charge* c_d is defined so that $S_L^{\mathrm{VB}} = \frac{c_d}{3} \log_2 L$
- *Uniform chains*: VB central charge $c_d \neq$ central charge
  $\Rightarrow$ VB entanglement entropy $\neq$ 'true' entanglement entropy.

Results: VB effective central charge



- *Valence-bond (effective) central charge* c_d is defined so that $S_L^{\mathrm{VB}} = \frac{c_d}{3} \log_2 L$
- *Uniform chains*: VB central charge $c_d \neq$ central charge
  $\Rightarrow$ VB entanglement entropy $\neq$ ‘true’ entanglement entropy.
- *Random chains*: VB effective central charge $c_d \approx \ln d$
  $\Rightarrow$ VB entanglement entropy $\approx$ ‘true’ entanglement entropy.

Conclusions

- The VB Monte Carlo method can be used to simulate chains of non-Abelian quasiparticles.
- *Random chains:* the "VB effective central charge" coincides with the "effective central charge": the $|GS\rangle$ is freezes into random singlet phase at long length scales.
- *Uniform chains:* the VB central charge for the uniform chain is different from the central charge $1 - 6/[(k + 1)(k + 2)]$.
- We confirm, for the first time, the "effective central charge" of the random $SU(2)_k$ chains ($\ln d$), and that of the Random Transverse Field Ising Model ($\frac{1}{2} \ln 2$) using the VB Monte Carlo method.