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MICROWAVE RESONANCE STUDY OF MELTING IN HIGH MAGNETIC FIELD WIGNER SOLID

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Wigner solids in two-dimensional electron systems in high magnetic field B exhibit a striking, microwave or rf resonance, that is understood as a pinning mode. The temperature, T_m, above which the resonance is absent, is interpreted as the melting temperature of the solid. Studies of T_m for many B and many sample densities n show that T_m is a function of the Landau level filling ν alone for a given sample. This indicates that quantum mechanics figures importantly in the melting. T_m also appears to be increased by larger sample disorder.

Keywords: Wigner crystal; fractional quantum Hall effect; pinning.

1. Introduction

Two-dimensional electron systems (2DES) become insulators in sufficiently large perpendicular magnetic fields (B). For 2DES with disorder low enough to exhibit the fractional quantum Hall effect (FQHE), this insulator terminates the series of FQHE states at low Landau filling ν, and in all cases we have looked at so far, exhibits a striking microwave or rf resonance. This resonance is a signature of a pinned Wigner solid, and is understood[1–4] as a collective, "pinning" mode, in which pieces of the solid oscillate within the pinning potential due the inevitable disorder

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within the sample. The Wigner solid, induced by a magnetic field, which suppresses the zero point motion of the electrons, was predicted[5] to be the ground state of a 2DES without disorder, even at high density, for Landau fillings ν less than around 1/6. When disorder is present, the pinning energy that the crystal gains can give it advantage[6] in competition with FQHE states, and raise the ν at which the pinned crystal becomes the ground state.

The pinning mode resonance disappears above a temperature, T_m, which we interpret as the melting temperature of the pinned Wigner crystal. A recent article [7] presented our systematic study of the dependence of T_m on the areal density, n, of the 2DES and on the magnetic field, B. The present paper reviews these results and provides some additional details of the data from which the results of ref. 7 were obtained.

2. Melting of the 2D Wigner Crystal

Research into the question of how the Wigner crystal melts has a long history. A two-dimensional Wigner crystal of classical electrons in zero magnetic field is a well understood system. Such a classical crystal was realized[8] long ago with electrons floating on the surface of liquid helium. Such systems are sufficiently dilute that they are nondegenerate even for temperatures low enough to produce Wigner crystallization. Their melting, signaled by the vanishing of a mode of the Wigner crystal-on-helium system, was in good agreement with the calculated values[9] of the classical melting temperature, $T_{cm} = e^2(n\pi)^{1/2}/4\pi\epsilon k_B\Gamma$, with $\Gamma \approx 130$ and ϵ the dielectric constant of the host. The increase of T_{cm} with n can be thought of as due to the increase of electron-electron interaction or, equivalently, of the moduli of the solid.

In principle, even a dense 2DES Wigner crystal is expected to behave classically in sufficiently high B. The size of a single particle electron wave function in lowest Landau level is the magnetic length $l_B = (\hbar/eB)^{1/2}$, and the ratio of the Wigner crystal lattice constant a to l_B determines the importance that quantum exchange and correlation can have on the properties of the system. This ratio is simply related to the filling factor, $\nu = nh/eB \propto (l_B/a)^2$, so that ν is a measure of the importance of quantum mechanics in the description of the solid. As a consequence the classical melting temperature of a Wigner crystal without disorder was expected to be recovered in the low ν limit.

It was realized early on[10,11] that quantum mechanics would play a role in the melting of the magnetically induced Wigner crystal. One early theoretical result [11] predicted that the melting temperature was predicted to be the $T_{cm}(n)$ multiplied by a ν dependent factor t_m, known as the reduced temperature. A number of early works[12] on magnetically induced Wigner solids in semiconductor-hosted 2DES, presented data in terms of this reduced temperature.

The main result[7] which we review here, is that that T_m is well-described as a function ν (or n/B) alone for a given sample. Since ν is a measure of the quantum

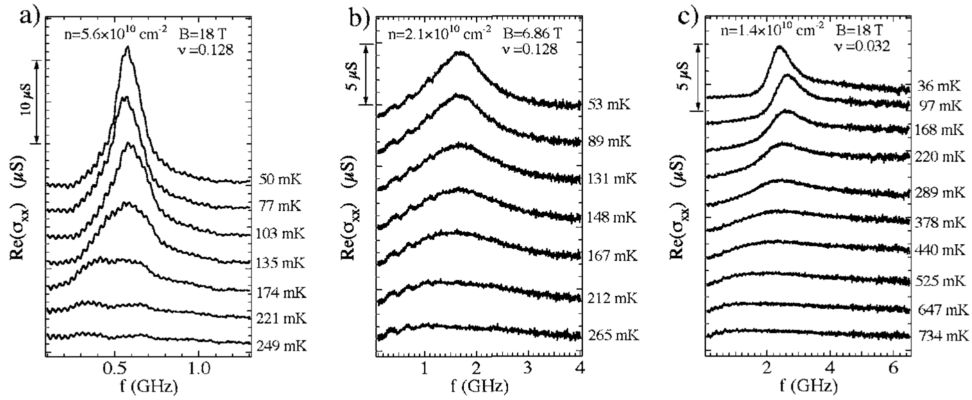

Fig. 1. Spectra, $\mathrm{Re}(\sigma_{xx})$ vs frequency, f, for sample 1, at many temperatures for different density (n) and magnetic field (B) combinations. Successive spectra are offset for clarity.

effects, the dependence of T_m on ν is an indication that quantum mechanics plays a central role in the melting of the low ν electron solid—and that its description in terms of the reduced temperature is in fact overly complicated. A further result is that $T_m(\nu)$ is larger in a sample with stronger disorder, as measured both by the mobility and by the frequency of the pinning mode, so that disorder appears to stabilize the solid phase.

3. Experimental

Microwave spectra, $\mathrm{Re}(\sigma_{xx})$ vs frequency f, were obtained as in other articles[1–4,7] from the measured loss of metal transmission lines patterned onto the front of the samples. The transmission lines are of a standard type known as coplanar waveguide [13], which has a central, driven conductor separated from broad side planes by slots of width $W \sim 30$ μm. In-plane electric field is well confined to the regions under the slots, in the high f, low conductivity ($|\sigma_{xx}|$) case relevant to the experiments. The 2DES, a fraction of a μm below the CPW, couples to it capacitively. We present $\mathrm{Re}(\sigma_{xx})$ data calculated from $\mathrm{Re}(\sigma_{xx}) = -W|\ln(P)|/2Z_0L$, where P is the transmitted power normalized to unity for vanishing σ_{xx}, $Z_0 = 50$ Ω is the characteristic impedance of the CPW calculated for $\sigma_{xx} = 0$, and L is the total length of the transmission line. We have performed calculations that account for the distributed coupling between CPW and 2DES and for reflections to ensure the validity of the formula under our measuring conditions. In all cases, we operated in the low power limit, in which spectra did not change on further reduction of the applied power.

4. Samples

We will be comparing data taken on two samples. Essential to our study, which requires independently variable B and n, a range of n is obtained for each sample

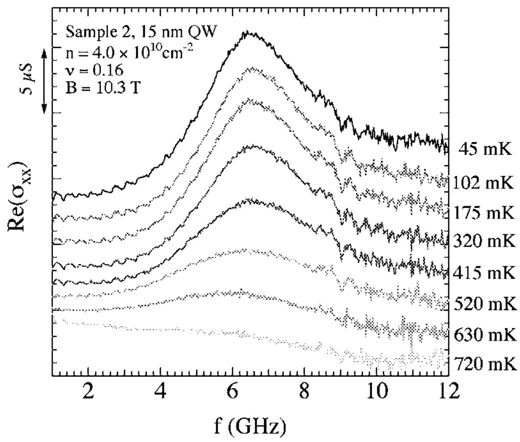

Fig. 2. Spectra, $\mathrm{Re}(\sigma_{xx})$ vs frequency f, for sample 2, at many temperatures. Successive spectra are offset for clarity.

by application of backgate bias. Sample 1 is a single heterojunction, for which n can be varied from 1.2 to 8.1×10^{10} cm^{-2}, with mobility 6 $\times10^{6}$ cm^2/V-s at the highest n. Sample 2 is a 15 nm-wide quantum well with n variable between 2.7 and 4.0×10^{10} cm^{-2}. Its high n mobility of 10^6 cm^2/V-s indicated disorder significantly larger than that of sample 1. Similar to a moderate disorder sample studied earlier[3], Sample 2 is insulating for ν lower than the range of the 1/3 FQHE, and its resonance is clearly present for $\nu \leq 0.28$.

5. Temperature Dependence of the Spectra

Fig. 1 shows spectra from Sample 1 at many temperatures, for three different states of n and B. the dependence of the spectra on n and B at low temperature has been covered in an earlier publication[2]. In each state, as temperature increases, the amplitude of the resonance gradually decreases, the resonance broadens, and there is a slight shift of the resonance toward lower frequency. The data in Fig. 1a and b were taken respectively with n = 5.6 and 2.1×10^{10}cm^{-2}, but with B adjusted to maintain the same $\nu = nh/eB = 0.128$. Though the resonance peak frequency f_{pk} is larger for the data in Fig. 1b, (mainly an effect of lower n, due to reduced electron-electron interaction[2,4]) it can be seen that the relative reduction in peak height σ_{pk} is similar at similar temperatures. The resonance of Fig. 1c measured for $n = 2.1 \times 10^{10}$cm^{-2}, and B = 18 T, has σ_{pk} proportionately less reduced by the elevated temperature, and clearly survives to higher temperature than that in either panel a or b. Comparing Fig. 1a and c shows the smaller n at fixed B produces a resonance that survives to higher temperature—exactly the opposite of what would be expected of a classical solid.

Fig. 2 shows the temperature dependence of resonance spectra from sample 2 at filling factor 0.16. As for sample 1, there is gradual reduction in σ_{pk} with increasing

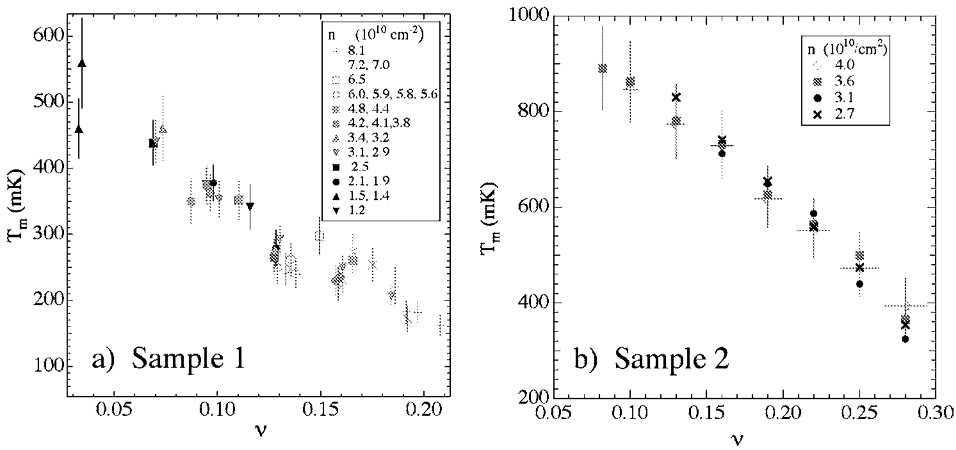

Fig. 3. Melting temperature T_m derived from the temperature dependence of the resonance vs Landau filling ν for a) Sample 1 and b) Sample 2. The densities n are listed in the legends.

temperature, with f_{pk} only shifting downward slightly. f_{pk} is larger than than any observed in Sample 1 for all n and B, consistent with the larger disorder in Sample 2, since the resonance is a pinning mode, in which larger disorder is expected to produce higher frequency. The resonance in Sample 2 survives to considerably higher temperatures than that in Sample 1.

6. Melting Temperature vs ν

We obtain melting temperatures, T_m, from temperature dependent spectra like those in Fig. 1 and Fig. 2, by extrapolating σ_{pk} vs temperature to zero, with σ_{pk} determined by Lorentzian fits. We estimate the uncertainty in T_m obtained this way as around 10%; estimating σ_{pk} by other means, such as simply taking the absolute maximum of the data, typically changes T_m by at most that amount.

Fig. 3a shows the plot of T_m vs ν for sample 1. Each of the points is from a set of temperature dependent spectra. Data from four cooldowns and 23 different densities n are included. Different symbols denote different ranges of n. It can be seen that the data group closely in a single curve $T_m(\nu)$, roughly to within the estimated error. Fig. 3b shows T_m vs ν for Sample 2. Data for four densitiesare each plotted with a different symbol. As for Sample 1, the data group tightly around a curve to within their error. T_m for Sample 2, which is more disordered, is significantly higher than that for Sample 1. Curves like those in Fig. 3a and b are reasonable representations of phase boundaries for individual samples, and depend on the disorder.

Fig. 4 presents the same data for the two samples as Fig. 3, but the reduced temperature, $t_m = T_m/T_{cm}(n)$ is plotted against ν. To facilitate comparison, the vertical (t_m) axes Fig. 4a and b span factors of 10 and 5, respectively the same as the T_m axes in Fig. 3a and b. The points in Fig. 4 often fall well outside the error

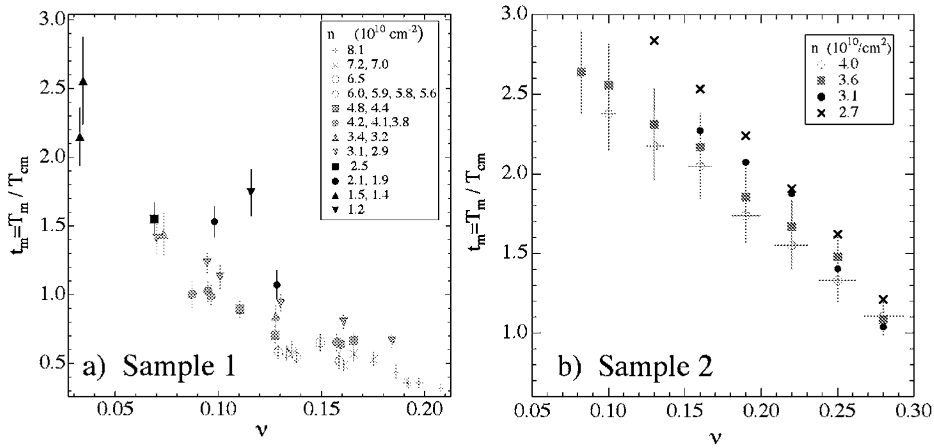

Fig. 4. Classical reduced temperature $t_m = T_m/T_{cm}$ (see text) vs Landau filling ν for a) Sample 1 and b) Sample 2. The densities n are listed in the legends.

bars from a single curve. More significantly, $t_m(\nu, n)$ is clearly and systematically increasing with n at any ν. The reduced temperature description of the melting thus does not work for data taken at different n, and is an inadequate means of plotting the melting curve as well as more complicated than simply taking $T_m(\nu)$.

7. Discussion

The data indicate that even down to $\nu \lesssim 0.1$, the classical description of melting, with n solely determining T_m is not correct. The simple extension of the classical using the reduced temperature is likewise not a good description of the data. Rather, dependence of T_m on ν is an unambiguous indication of the importance of quantum mechanical effects in the melting.

Another main result is the importance of disorder associated with an individual sample in determining $T_m(\nu)$. The well-grouped T_m vs ν plots in Fig. 3, obtained by backgating samples with quite different structures, indicate that any change with backgate bias of the effective disorder, due to compression or motion of the vertical wave function, is significantly less important than the change of ν.

The increase of T_m with disorder is an unusual feature of the melting. One possibility is that increased disorder favors the pinned solid, in which the electron positions adjust to minimize[6] the energy. Comparison with other systems is also of interest, and an increase in melting temperature has been observed[14] for high T_c superconductors with artificial columnar defects. Classical melting with such defects is analogous to quantum melting according to comments by Giamarchi[15]. Melting temperature decreasing with larger disorder, opposite to the behavior reported here, is more common, and is seen for example in helium[16] in pores and in high T_c superconductors with point defects[17].

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